OXFORD IB DIPLOMA PROGRAMME

END OF CHAPTER TESTS

MATHEMATICS: ANALYSIS AND APPROACHES

HIGHER LEVEL

COURSE COMPANION

ENHANCED ONLINE

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OXFORD

1 From patterns to generalizations: sequences and series

Section A. A calculator is not allowed

1 Expand and simplify $(5-3x)^4$.

2 Determine the first three terms in the expansion of $(1-2x)^5(1+x)^7$ in ascending powers of x.

3 The sum of the first n terms of a geometric sequence is $S_n = 1-\left(\frac{a}{3}\right)^n$

Find and expression for,

a the first term of the sequence

b the common ratio of the sequence.

Consider the sum to infinity of the sequence.

c Determine the values of a such that the sum to infinity exists.

d Find the sum to infinity when it exists.

4 An arithmetic sequence of n terms has $S_n = 90,\ u_1 = 1.5 and u_n = 7.5$

Find

a the common difference

b the value of n.

Section B. A calculator is allowed

5 Consider the arithmetic sequence 8, 26, 44, ...

a Find an expression for the nth term.

b Write down the sum of the first n terms using sigma notation.

c Calculate the sum of the first 15 terms.

6 A geometric sequence has a first term of 2 and a common ratio of 1.05. Find the value of the smallest term that is greater than 500.

7 Ron invests $5000 at a compound interest rate of 6.3 % p.a.

a How much money will Ron have at the end of five years?

Ron will leave his money in the bank until he has more than $10 000.

b How many full years will Ron have to wait?

8 An ice cream sundae menu has six flavours and 4 toppings. How many different types of sundae can you order if you want two different flavours and 1 topping?

9 Top school has 6 periods in a school day. In how many ways can they organize 5 subjects so that each subject is allowed at least one period?

10 In an arithmetic sequence, $S_6 = 81$ and $S_{11} = 231$

Find

a the first term

b the common difference.

The sum of the first two terms of a geometric series is 1 and the sum of its first four terms is 5. If all of its terms are positive, find

c the first term and

d the common ratio.

The *n*th term of a new series is defined as the product of the *n*th term of the arithmetic series and the *n*th term of the geometric series above.

e Show that the *n*th term of this new series is $(n+1)(2^{n-1})$.

f Using mathematical induction, prove that $\sum_{1}^{k}(n+1)(2^{n-1}) = k2^k, k \in \mathbb{Z}^+$

Answers

1 $1(5^4)+4(5^3)(-3x)+6(5^2)(-3x)^2+4(5)(-3x)^3+1(-3x)^4$

$=625-1500x+1350x^2-540x^3+81x^4$

2
$(1-2x)^5(1+x)^7 \Rightarrow (1+5(-2x)+10(-2x)^2)\times(1+7x+21x^2) \Rightarrow 1+7x+21x2-10x-70x2+40x2=1-3x-9x^2$

3 **a** $u_1=S_1=1-\left(\frac{a}{3}\right)^1=1-\frac{a}{3}$

b $u_2=S_2-S_1=\left(1-\left(\frac{a}{3}\right)^2\right)-\left(1-\frac{a}{3}\right)=\frac{a}{3}\left(1-\frac{a}{3}\right)$

$r=\frac{a}{3}$

c Sum to infinity when $.\ 0<\frac{a}{3}<1\ ..\ \therefore 0<a<3$

d $S_\infty=\frac{u_1}{1-r}=\frac{1-\frac{a}{3}}{1-\frac{a}{3}}=1$

4 **a** $S_n=\frac{n}{2}(u_1+u_n)\ .\ 90=\frac{n}{2}(1.5+7.5)\ n=20$

b $u_n=u_1+(n-1)d$

$7.5=1.5+19d$

$d=\frac{6}{19}$

5 **a** $u_n=u_1+(n-1)d=8+(n-1)18=18n-10$

b $\sum_1^n 18n-10$

c $S_{15}=\frac{15}{2}((2\times 8)+(15-1)18)=2010$

6 $2\times 1.05^{n-1}>500$

$n-1>\frac{\log 250}{\log 1.05}$ (or a graphical solution)

$n-1>113.1675$

$n=115$

$u_{115}=521$

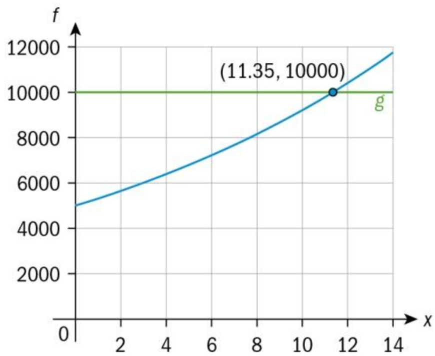
7 **a** $FV = 5000(1.063)^5 = \$6786$

b $5000(1.063)^n > 10000$

Graphical, log or table function solution

$n > 11.345 \, years$

Ron has to wait 12 years.



8 $C_2^6 \times C_1^4 = 15 \times 4 = 60$

9 Total number of arrangements $= \dfrac{P_5^6 \times C_1^5}{2!} = 1800$

10 $S_n = \dfrac{n}{2}(2u_1 + (n-1)d$

$81 = \dfrac{6}{2}(2u_1 + 5d)$ and $231 = \dfrac{11}{2}(2u_1 + 10d)$

$2u_1 + 5d = 27$ $u_1 + 5d = 21$

a $u_1 = 6$

b $d = 3$

c $u_1 + u_1 r = 1$

$u_1(1 + r) = 1$

$1 + r = \dfrac{1}{u_1}$

$u_1 + u_1 r + u_1 r^2 + u_1 r^3 = 5$

$(u_1 + u_1 r) + u_1 r^2 (1 + r) = 5$

$(1 + u_1 r^2) \times \dfrac{1}{u_1} = 5$

$r = 2$

d $u_1(1+2) = 1$

$u_1 = \frac{1}{3}$

e Arithmetic series $u_n = 3n+3$

Geometric series $u_n = \frac{1}{3}\left(2^{n-1}\right)$

$3(n+1)\times\frac{1}{3}\left(2^{n-1}\right) = (n+1)\left(2^{n-1}\right)$

f Prove $P_k : \sum_1^k (n+1)\left(2^{n-1}\right) = k2^k, k \in \mathbb{Z}^+$

Show it is true for $k = 1$

$k = 1, (1+1)\left(2^{1-1}\right) = 1(2^1)$

Now consider $k = r + 1$

$$\sum_1^{r+1}(n+1)\left(2^{n-1}\right) = r2^r + (r+1)2^r = 2^r(2r+2) = 2(r+1)2^r = (r+1)\left(2^{r+1}\right)$$

Therefore, it is true for $k = r + 1$

P_{r+1} is true when P_r is true.

P_1 is true.

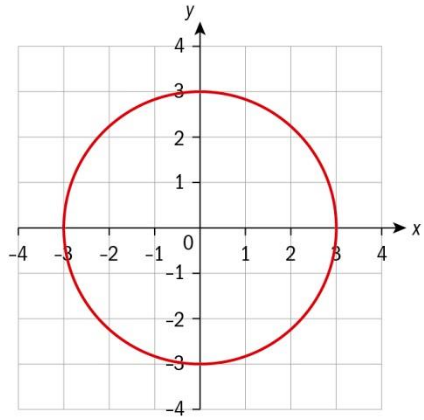
Therefore, P_k is true .

2 Representing relationships: introducing functions

A calculator is allowed

1 Which of the following relations are functions?

a



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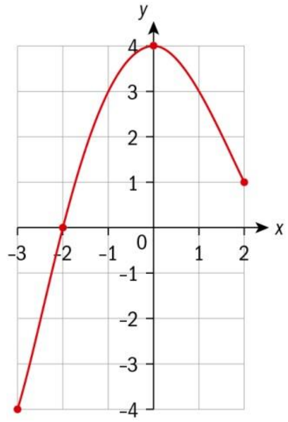

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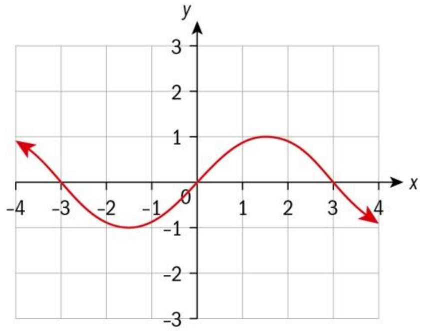

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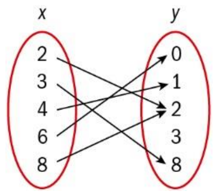


2 State the range and domain for each relation or function.

3 Let $f(x) = \dfrac{4}{x+2}$ and $g(x) = x - 1$.

Find

a $(g \circ f)(x)$

b $(g \circ f)^{-1}(x)$

4 $f(x) = \dfrac{2x-3}{x-1}$

Find an expression for $f^{-1}(x)$.

5 A function is defined as $f(x) = a\sqrt{x}$, with $a > 0$ and $x \geq 0$.

a Sketch the graph of $y = f(x)$.

b Explain why f is a one-to-one function.

c Find $f^{-1}(x)$

d If the graphs of $f(x)$ and $f^{-1}(x)$ intersect at the point (9, 9) find the value of *a.*

6 **a** Write the equation $f(x) = 3x^2 - 6x + 5$ in the form $f(x) = a(x-h)^2 + k$

b Describe the transformations that transformed $f(x) = x^2$ into $f(x) = 3x^2 - 6x + 5$

7 Solve $|2x| \leq |x-3|$

8 Use an algebraic method to determine whether the following functions are even odd or neither

a $f(x) = x^4 - 5x^2 + 1$

b $g(x) = x^5 - 3x^3 + 4$

c $h(x) = \dfrac{x^2+1}{x^3-x}$

9 Express $\dfrac{3x+5}{2x^2-5x-3}$ as the sum of two fractions with linear denominators.

Answers

1 b, c and d

2 **b** $-3 < x \le 2, -4 < y \le 4$

c $x \in \mathbb{R}, -1 \le y \le 1$

b Domain $\{2,3,4,6,8\}$, Range $\{0,1,2,3,8\}$

3 **a** $(g \circ f)(x) = \frac{4}{x+2} - 1$

b $x = \frac{4}{y+2} - 1$

$x + 1 = \frac{4}{y+2}$

$y + 2 = \frac{4}{x+1}$

$y = \frac{4}{x+1} - 2$

4 $y = \frac{2x-3}{x-1}$

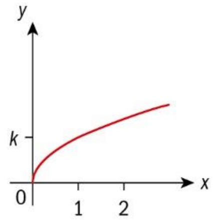
$x = \frac{2y-3}{y-1}$ $x(y-1) = 2y-3$

$xy - x = 2y - 3$ $xy - 2y = x - 3$

$y(x-2) = x - 3$

$f^{-1}(x) = \frac{x-3}{x-2}$

5 **a**



b A horizontal line will only cross *f(x)* once.

c $x = a\sqrt{y}$

$\frac{x}{a} = \sqrt{y}$

$y = \frac{x^2}{a^2}$

d $a\sqrt{x} = \frac{x^2}{a^2}$

$a\sqrt{9} = \frac{81}{a^2}$

$a^3 = 27$

$a = 3$

6 **a** $3x^2 - 6x + 5 = 3(x^2 + 2x) + 5 = 3(x-1)^2 + 2$

b A vertical stretch scale factor 3 and a translation $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

7 $4x^2 \le x^2 - 6x + 9$

$3x^2 + 6x - 9 \le 0$

$(x-1)(x+3) \le 0$

$x \ge -3, x \le 1$

8 **a** Replacing x with -x we obtain: $f(-x) = (-x)^4 - 5(-x)^2 + 1 = x^4 - 5x^2 + 1 = f(x)$.

So $f(x)$ is even.

b $g(-x) = (-x)^5 - 3(-x)^3 + 4 = -x^5 + 3x^3 + 4 \ne g(x)$.

$g(-x)$ is not equal to $g(x)$ or $-g(x)$, so $g(x)$ is neither even nor odd.

c $h(-x) = \frac{(-x)^2 + 1}{(-x)^3 - (-x)} = \frac{x^2 + 1}{-x^3 + x} = -h(x)$

$h(x)$ is odd

9 $\frac{3x+5}{2x^2 - 5x - 3} = \frac{3x+5}{(2x+1)(x-3)} = \frac{A(x-3)}{2x+1} + \frac{B(2x+1)}{x-3}$

$3x + 5 = A(x-3) + B(2x+1)$

$3x + 5 = Ax - 3A + 2Bx + B$

$3 = A + 2B$ ....#1

$5 = -3A + B$

$10 = -6A + 2B \ldots..\#2$

$\#2 - \#1$

$7 = -7A$

$A = -1$

Substitute into #1

$3 = -1 + 2B$

$B = 2$

$$\frac{3x+5}{2x^2-5x-3} = \frac{2}{x-3} - \frac{1}{2x+1}$$

3 Expanding the number system: complex numbers

Section A. A calculator is not allowed

1 Use the discriminant to determine the nature of the roots of $x^2 - 2x + 6 = 0$

2 Given $8x^2 - 2x = 3$ without solving, find

a the sum of the roots

b the product of the roots.

3 Simplify

a $\sqrt{-16}$ **b** $\sqrt{-49}$ **c** $\sqrt{-27}$

Express in the form of a complex number $a + bi$

d $(2 + 3i) + (-4 + 5i)$

e $(2 - i)(3 + i)$

f $(5 + 3i)^2$

g $\frac{5}{3-\sqrt{2}}$

h $\frac{8+4i}{1-i}$

4 Given that $(a + bi)^2 = 3 + 4i$

a Find a pair of simultaneous equations involving a and b.

b Hence find the two square roots of $3 + 4i$.

5 Solve the equation $5z + \, = 6 - 18i$, where z^* is the conjugate of z, when $|z| = \sqrt{10}$.

6 Solve the simultaneous equations

$$\mathrm{i}\,z_1 + 2z_2 = 3$$
$$z_1 + (1 - \mathrm{i})\,z_2 = 4$$

giving z_1 and z_2 in the form a + bi, where x and y are real.

7 Consider the polynomial $f(x) = x^4 + ax^3 + bx^2 + cx + d$, where a, b, c, d $\mathbb{R}$.

Given that 1 + i and 1 − 2i are zeros of $f(x)$, find the values of a, b, c and d.

Section B. A calculator is allowed

8 **a** Show that $(x-3)$ is a factor of x^3-2x^2-5x+6 and find the other two factors.

b Use your GDC to help sketch $f(x)$

c Write down the solution to the inequality $x^3-2x^2-5x+6<0$

9 **a** Write down the x intercept of $f(x)=2x^3-6x^2+7x$.

b Find the exact roots of f

Answers

1 $b^2 - 4ac = (-2)^2 - (4 \times 1 \times 6) = -20.$

Two distinct, imaginary roots.

2 Rearrange to $8x^2 - 2x - 3 = 0$

a $sum = -\left(\frac{-2}{8}\right) = \frac{1}{4}$

b $product = -\frac{3}{8}$

3 **a** $\sqrt{-16} = \sqrt{(16)(-1)} = 4i$

b $\sqrt{-49} = \sqrt{(49)(-1)} = 7i$

c $\sqrt{-27} = \sqrt{(27)(-1)} = 3i\sqrt{3}$

d $(2+-4)+(3i+5i) = (2+3i)+(-4+5i) = -2+8i$

e $(2-i)(3+i) = 6+2i - 3i - i^2 = 6 - i - (-1) = 7 - i$

f $(5+3i)^2 = 25+30i+9i^2 = 25+30i+9(-1) = 16+30i$

g $\frac{5}{3-\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{15+5\sqrt{2}}{7}$

h $\frac{6+2i}{1-i} \times \frac{1+i}{1+i} = \frac{6+6i+2i+2i^2}{1+i-i-i^2} = 2+4i$

4 **a** $a^2 + 2iab - b^2 = 3 + 4i$

Equating the real and imaginary parts
$a^2 - b^2 = 3,\ 2ab = 4f(x) = x^3 - 2x^2 - 5x + 6.$

$b = \frac{2}{a}$

$a^2 - \left(\frac{2}{a}\right)^2 = 3$

$a^4 - 3a^2 - 4 = 0 \qquad (a^2 - 4)(a^2 + 1) = 0$

$a = \pm 2$ and $b = \pm 1$

b $\sqrt{3+4i} = 2 + i, -2 - i$

5 $5zz^* + 10 = (6 - 18i)z^*$

Letting $z = a + ib$
$5 \times 10 + 10 = (6 - 18i)(a - bi)$

$60 = 6a - 6bi - 18ai - 18b$
Equating real and imaginary parts
$6a - 18b = 60$ and $6b + 18a = 0$
$a = 1$ and $b = -3$
$z = a + bi = 1 - 3i$

6

$iz_1 + 2z_2 = 3 \Rightarrow z_2 = -\frac{1}{2}iz_1 + \frac{3}{2}$
$z_1 + (1-i)z_2 = 4$
$z_1 + (1-i)(-\frac{1}{2}iz_1 + \frac{3}{2}) = 4$
$z_1 + (-\frac{1}{2}iz_1 + \frac{3}{2} + \frac{1}{2}i^2z_1 - \frac{3}{2}i) = 4$
$\frac{1}{2}z_1 - \frac{1}{2}iz_1 = 4 - \frac{3}{2} + \frac{3}{2}i$
$z_1 - iz_1 = 5 + 3i$

Letting $z_1 = x + iy$

$$x + iy - ix - i^2y = 5 + 3i$$

Equate real and imaginary parts

$x + y = 5$
$-x + y = 3$
$2y = 8$
$y = 4$
$x = 1$
$\therefore z1 = 1 + 4i$

$z_2 = -\frac{1}{2}iz_1 + \frac{3}{2}$
$z_2 = -\frac{1}{2}i(1 + 4i) + \frac{3}{2}$
$z_2 = -\frac{1}{2}i - 2i^2 + \frac{3}{2}$

$\therefore z_2 = \frac{7}{2} - \frac{1}{2}i$

7 $1+i$ is a zero $\therefore$ its conjugate $1 - i$ is also a zero
$1 - 2i$ is a zero $\therefore$ its conjugate $1 + 2i$ is also a zero
$\left(x - (1 - i)\right)\left(x - (1+i)\right) = x^2 - 2x + 2$ $\left(x - (1 - 2i)\right)\left(x - (1+2i)\right) = x^2 - 2x + 5$
$f(x) = \left(x^2 - 2x + 2\right)\left(x^2 - 2x + 5\right) = x^4 - 4x^3 + 11x^2 - 14x + 10$

$a = -4, b = 11, c = -14, d = 10$

8 a

$(x - 3)$ is a factor if $f(3) = 0$.

$f(3) = (3)^3 - 2(3)^2 - 5(3) + 6 = 0.$

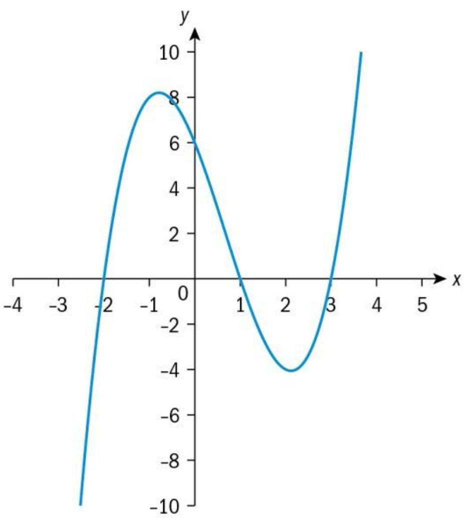
$\therefore x - 3$ is a factor

To find the other factors, divide $x^3 - 2x^2 - 5x + 6.$ by $(x - 3)$

$$
\begin{array}{r}
x^2 + x - 2 \\
x - 3 \overline{) x^3 - 2x^2 - 5x + 6} \\
\underline{x^3 - 3x^2} \qquad\qquad \\
x^2 - 5x \qquad \\
\underline{x^2 - 3x} \qquad \\
-2x + 6 \\
\underline{-2x + 6} \\
0
\end{array}
$$

$f(x) = x^3 - 2x^2 - 5x + 6 = (x-3)(x^2 + x - 2) = (x-3)(x-1)(x+2)$

b



c Less than zero where the curve is below the x axis.

$x < -2, 1 < x < 3$

9 **a** $(0,0)$

b Using $2x^2 - 6x + 7 = 0$

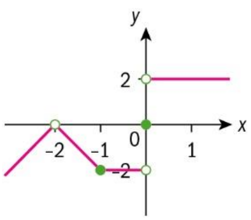
$$x = \frac{-(-6) \pm \sqrt{(-6)^2 - (4 \times 2 \times 7)}}{2(2)} = \frac{6 \pm \sqrt{-20}}{4}$$

Roots are $0, \frac{3 + i\sqrt{5}}{2}, \frac{3 - i\sqrt{5}}{2}$

4 Measuring change: differentiation

Section A. A calculator is not allowed

1 The graph of a function $y = f(x)$ is shown below.



Use the graph to find the following limits, if they exist. If a limit does not exist, explain why.

a $\lim_{x\to -2} f(x)$ **b** $\lim_{x\to -1} f(x)$ **c** $\lim_{x\to 0} f(x)$

2 Find the following limits, if they exist.

a $\lim_{x\to 2} x^3$ **b** $\lim_{x\to 4}\left(\frac{x^2-16}{x-4}\right)$ **c** $\lim_{x\to\infty}\left(\frac{3x^3-x+1}{x^3+x^2}\right)$

3 Find the value b that makes the function continuous.

$$f(x) = \begin{cases} \frac{1}{2}x - 2, & x \geq 1 \\ b - x, & x < 1 \end{cases}$$

4 Differentiate $y = 3x^2 - 2x + 1$ from first principles.

5 Find the equation of the tangent and normal to $y = 3x^2 - x^3$ at $(1,2)$.

6 Differentiate

a $(x^3 + 1)^{10}$ **b** $x^2(x + 1)^5$ **c** $\frac{x}{1+x^2}$

7 If $x^2 - xy + y^2 = 5$, find the value of y'.

8 Find the gradient of the normal to the curve $3x^2y + 2xy^2 = 2$ at $(1, -2)$

Section B. A calculator is allowed

9 The area of a circle is growing so that after *t* seconds the area is modelled by $A = 3t^2 + \frac{1}{5}t$. Find the rate of increase after 2.65 seconds.

10 Consider the function $f(x) = 3x^4 - 4x^3 - 2$

a Find all turning points, and determine their nature (you should justify your answers).

b Find the coordinates and nature of any inflexion points.

c Sketch the graph of *f*, with $-1 \le x \le 2 \text{ and } -4 \le \text{y} \le 8$ labelling a horizontal inflexion A, minimum point B and a non-horizontal inflexion C.

11 The function $f(x) = \frac{10(x-1)}{x^2}$ has $f'(x) = -\frac{10(x-2)}{x^3}$ and $f''(x) = \frac{20(x-3)}{x^4}$

a Find the zeros of $f(x)$.

b The coordinates of the local maximum point.

c The intervals where $f(x)$ is concave up.

Given $f(x)$ has the x and y axes as asymptotes,

d Sketch the function for $x \ge 0$

Answers

1 **a** 0 **b** -2

c Does not exist. The graph jumps as it approaches both -2 and 2 as $x \to 0$

2 **a** 8

b $\lim\limits_{x\to 4}\left(\dfrac{x^2-16}{x-4}\right) = \lim\limits_{x\to 4}\left(\dfrac{(x+4)(x-4)}{x-4}\right) = \lim\limits_{x\to 4}(x+4) = 8$

c $\lim\limits_{x\to \infty}\left(\dfrac{3x^2-x+1}{x^3+x^2}\right) = \lim\limits_{x\to \infty}\left(\dfrac{3-1/x^2+1/x^3}{1+\dfrac{1}{x}}\right) = \dfrac{3}{1} = 3$

3 $\dfrac{1}{2}x + 2 = b - x$

$\dfrac{1}{2}(1) + 2 = b - 1$

$b = -\dfrac{1}{2}$

4 $y + \delta y = 3(x+\delta x)^2 - 2(x+\delta x) + 1 = 3x^2 + 6x\delta x + 3(\delta x)^2 - 2x - 2\delta x + 1$

Subtracting $y = 3x^2 - 2x + 1$ gives $\delta y = 6x\delta x + 3(\delta x)^2 - 2\delta x$

Dividing by δx

$\dfrac{\partial y}{\partial x} = 6x + 3(\partial x) - 2$

As $\delta x \to 0$, we have $\dfrac{\partial y}{\partial x} = 6x + 2$

5 $\dfrac{dy}{dx} = 6x - 3x^2$

At (1,2), $\dfrac{dy}{dx} = 6(1) - 3(1)^2 = 3$

The equation of the tangent is $y - 2 = 3(x-1)$ or $y = 3x - 1$

The equation of the normal is $y - 2 = -\dfrac{1}{3}(x-1)$ or $x + 3y = 7$

6 **a** Let $u = x^3 + 1$, then $y = u^{10}$

$\dfrac{du}{dx} = 3x^2$, $\dfrac{dy}{du} = 10u^9$

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx} = 10\left(x^3 + 1\right)^9 \times 3x^2 = 30x^2\left(x^3 + 1\right)^9$$

b $\frac{dy}{dx} = \left(x^2\right)5\left(x+1\right)^4 + 2x\left(x+1\right)^5 = x\left(x+1\right)^4\left(7x+2\right) = x\left(7x+2\right)\left(x+1\right)^4$

c $\frac{x\left(1+x^2\right) - 2x^2}{\left(1+x^2\right)^2} = \frac{1-x^2}{\left(1+x^2\right)^2}$

7 $x^2 - xy + y^2 = 5$

$$\frac{d}{dx}(x^2 - xy + y^2) = \frac{d}{dx}(5)$$

$$\frac{d}{dx}(x^2) - \frac{d}{dx}(xy) + \frac{d}{dx}(y^2) = 0$$

$$2x - x\frac{dy}{dx} - y + 2y\frac{dy}{dx} = 0$$

$$(-x + 2y)\frac{dy}{dx} = -2x + y$$

$$\frac{dy}{dx} = \frac{y - 2x}{2y - x}$$

$$y' = \frac{y - 2x}{2y - x}$$

8 $6xy + 3x^2\frac{dy}{dx} + 2y^2 + 4xy\frac{dy}{dx} = 0$

Letting $x = 1$and $y = -2$

$$-12 + 3\frac{dy}{dx} + 8 - 8\frac{dy}{dx} = 0$$

$$-5\frac{dy}{dx} = 4$$

$$\frac{dy}{dx} = -\frac{4}{5}$$

Gradient of the normal is $\frac{5}{4}$

$$y - (-2) = \frac{5}{4}(x - 1)$$

$$y = \frac{5}{4}x - \frac{13}{4}$$

9 $\frac{dA}{dt} = 6t + 0.125 = 6(2.65) + 0.125 = 16.025 cm^2 s^{-1}$

10 **a** $f'(x) = 12x^3 - 12x^2 = 12x^2(x-1)$

$f'(x) = 0$, when $x = 0, 1$.

When $x = 0$

$f''(x) = 36x^2 - 24x$

$f''(0) = 36(0)^2 - 24(0) = 0$

The y value is $f(0) = 3(0)^4 - 4(0)^3 - 2 = -2$.

Horizontal inflexion point at $(0, -2)$

When $x = 1$

$f''(1) = 36(1)^2 - 24(1) = 12$

The y value is $f(1) = 3(1)^4 - 4(1)^3 - 2 = -3$.

Minimum point at $(1, -3)$

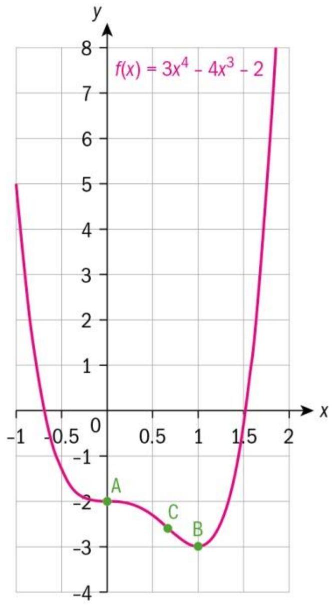
b $f''(x) = 0$ at an inflexion point.

$f''(x) = 36x^2 - 24x = 12x(3x-2) = 0$ when $x = 0, \frac{2}{3}$

$f\left(\frac{2}{3}\right) = 3\left(\frac{2}{3}\right)^4 - 4\left(\frac{2}{3}\right)^3 - 2 = -\frac{70}{27}$ or -2.59.

There is a non-horizontal point of inflexion at $\left(\frac{2}{3}, 2.59\right)$

c



11 **a** $10(x-1)=0$ when $x=1$

b Maximum point where $f'(x)=0$, $-10(x-2)=0, x=2$

It is a maximum point when $f''(x)<0. f''(2)=\dfrac{20(2-3)}{2^4}=-\dfrac{5}{4}$.

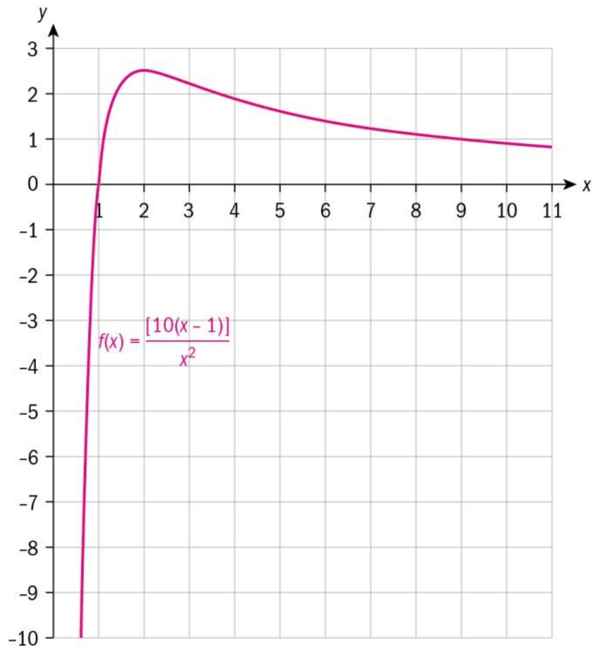
The y value is $f(2)=\dfrac{10(2-1)}{2^2}=2.5$

The local maximum point is $(2, 2.5)$.

c $f(x)$ is concave up where $f''(x)>0$

$20(x-3)>0, x>3$

d



5 Analysing data and quantifying randomness: statistics and probability

Section A. A calculator is not allowed.

1 Evan is studying the average weight of fish caught at a port.

Choose from convenience, simple random, systematic, stratified or quota to classify each of the following sampling techniques that Evan might use.

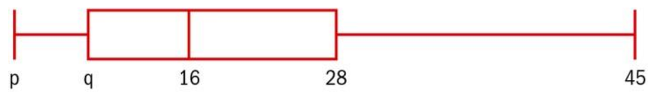
a Evan selects any 100 fish at random.

b A random fish is chosen and then every 20^{th} fish after that until Evan has 100 fish.

c The fish consist of 80% round fish and 20% flat fish. Evan selects 80 round and 20 flat fish.

d A sample of 100 fish is taken by organizing the fish by 5 species and then taking 20 from each species

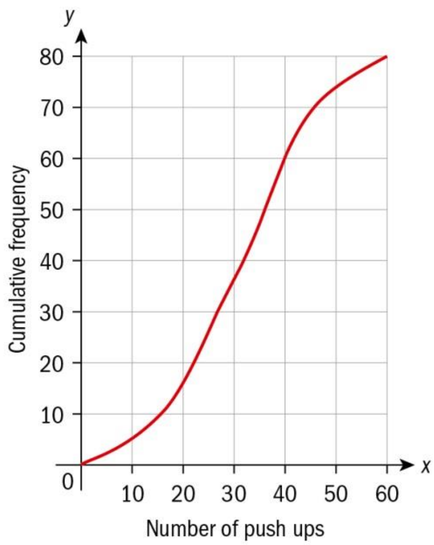
2 This diagram is a box plot for a data set.



a Write down the median.

b If the range is 42 and the interquartile range if 16, find the values of p and q.

3 The cumulative frequency diagram shows the number of push ups done in one minute by a group of 80 athletes.

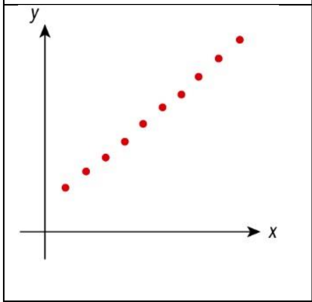
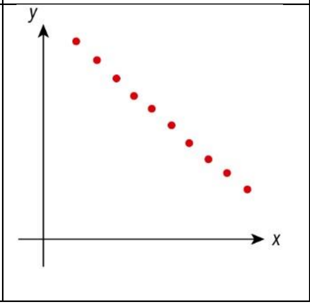
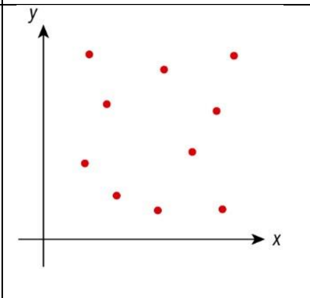
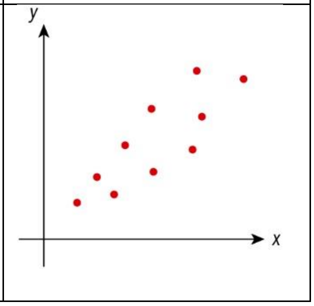


a Write down the median.

b Find the interquartile range

c The top 10% of athletes completed at least how many push ups?

4 Match the diagram with the correlation coefficient:

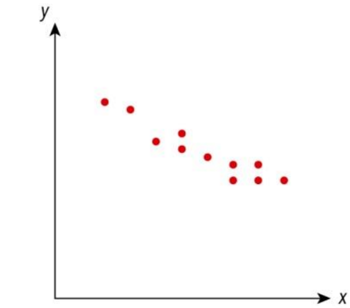
A	B	C	D
			

i -1 **ii** 0 **iii** 0.8 **iv** 1

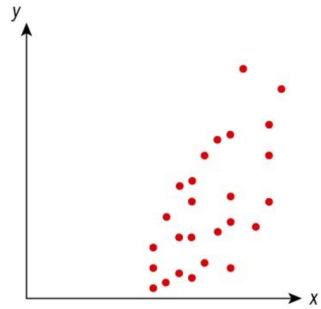
5 Describe the correlation for each diagram using the words:

Strong, moderate weak, positive, negative, linear, non linear, strong, moderate weak, no correlation.

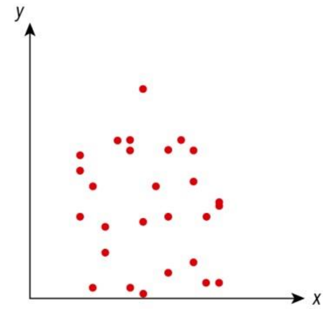
A



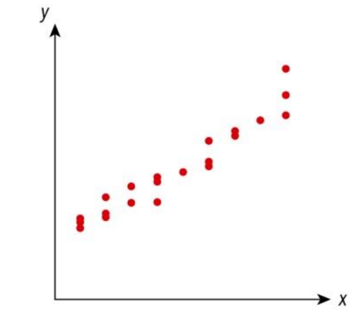
B



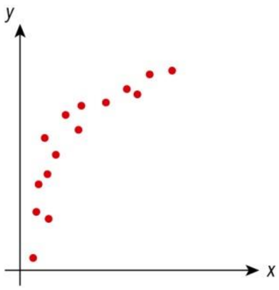
C



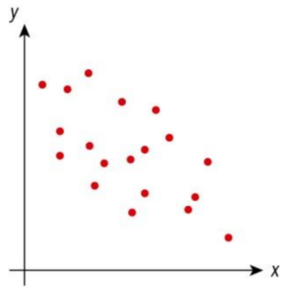
D



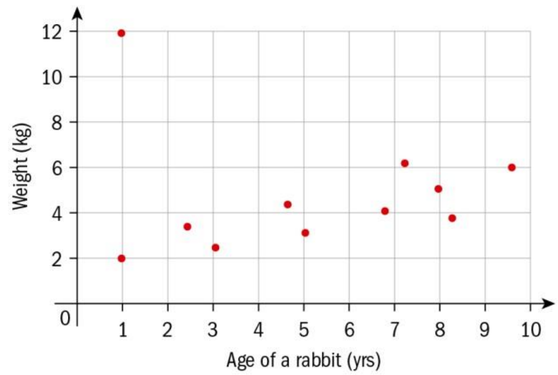
E



F



6 The scatter diagram shows the age of a family of pet rabbits and their weights in kg.



a Describe the correlation.

b What should you do with the outlier? Explain your reasons.

7 The production cost at Luigi's pizzeria is modelled by

$c(p) = 6p + 50$

where c is the production cost in dollars and *p* is the number of pizzas made.

a How much does it cost Luigi if the shop does not produce any pizzas?

b Interpret the meaning of the value of the gradient, 6.

8 A basketball coach recorded the average number of training hours per week for him team and the average change in the number of points scored in a season.

Hours (x)	0	1	2	3	4	5
Points change (y)	-5	0	1	10	15	18

a Show the data on a scatter plot.

b Find the mean point and indicate this on your scatter plot by the label M.

c Draw a line of best fit through your data.

d Describe the correlation.

e What can you say about the number of hours trained and the number of points scored?

Section B. A calculator is allowed

9 Khabib has a papaya orchard. On a Sunday he harvests 72 green papaya and 28 yellow. The mean weight of a red papaya is 1.79kg and the mean weight of a yellow papaya is 1.62kg.

Find the mean weight of Khabib's 100 papayas.

10 Here is a group of university friends and their ages

Age (yrs)	19	20	21	22	23
F	6	8	7	11	9

Find

a the mean

b the standard deviation

c the variance.

11 Ten students recorded how many hours of exercise they have at the weekend and their weights in kg.

Hours (x)	6	2	7	1	0	3	10	8	9	4
Weight(y)	80	60	70	50	90	80	70	100	55	60

Write down:

a the equation of the regression line.

b the r value

c Is it appropriate to estimate the weight of a student who does 5 hrs of exercise at the weekend? Give reasons.

12 Here are the scores for 10 of the 12 students in my class for mathematics and science.

Mathematics(x)	90	66	84	75	90	88	69	95	73	81
Science(y)	73	60	79	67	78	67	55	82	59	80

a Write down the r value.

b What does this tell you about the correlation

c Write down the equation of the y on x regression line.

Sara was absent for the science test but scored 80 in mathematics.

d Estimate her score using the equation of the regression line.

e Is this a valid estimate? Give reasons.

f Can the regression line be used to estimate the score of a student who scored 10% in mathematics? Give reasons.

g Find the equation of the regression line of x on y.

h What can this equation be used for?

13 I asked the first ten people that I saw this morning "How many pairs of shoes do you own"

Age (x)	15	24	42	13	56	16	14	20	6	12
Pairs of shoes (y)	2	7	5	4	6	8	4	8	2	6

a What type of sampling method did I use?

b Write down the r value.

c Explain what a positive value for the coefficient of correlation indicates.

d Write down the linear regression equation of y on x in the form $y = ax + b$

e Use your equation to determine the number of pairs of shoes that an 18yr old would have.

f Can your answer in part e be considered reliable? Give a reason for your answer.

14 Vin thinks that price of a car is related to its age and collects the following data where the age of the car is in years and the cost is in thousands of dollars.

Age of the car(x)	0	5	10	15	20	25	30	35
Cost ($1000) ($y$)	20	15	12	8	1	6	13	18

a Show this data on a scatter plot.

b Can you draw a single line of best fit to represent this data? Explain your answer.

c Find two piecewise linear functions that best represent the data.

Answers

1 **a** simple random **b** systematic **c** quota **d** stratified.

2 **a** 16

b $p = 45 - 42 = 3$ $q = 28 - 16 = 12$

3 **a** The median is 32

b $IQR = Q_3 - Q_1 = 40 - 22 = 18$

c 10% of 80= 8.

The 72nd athlete completed 48 push ups.

4 A 1 B -1 C 0 D 0.8

5 **a** Strong positive linear

b Weak positive linear

c No correlation

d Moderate positive linear

e Non linear.

f Weak, negative linear

6 **a** Moderate positive correlation

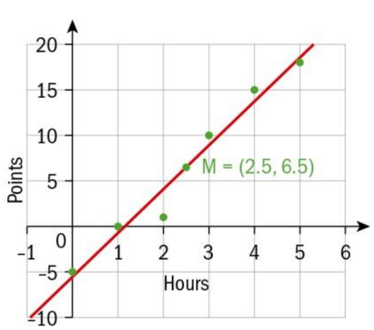
b Discard the outlier. It appears to be an error.

7 **a** \$50

b Each pizza costs \$6 to produce.

8 **a**, **b**, **c** on the graph.

b $\left(\bar{x},\bar{y}\right) = \left(\dfrac{0+1+2+3+4+5}{6}, \dfrac{-5+0+1+10+15+18}{6}\right) = (2.5, 6.5)$



d There is a strong, positive linear correlation.

e As the number of hours of practice increases the number of points scored increases.

9 $Mean = \frac{(72 \times 1.79) + (28 \times 1.62)}{100} = 1.74kg$

10 **a** $\bar{x} = \frac{\sum fx}{\sum f} = \frac{870}{41} = 21.2$

b 1.37

c $thevariance = (1.370636\ldots)^2 = 1.88$

d 21.2+x

e 1.37

11 **a** $y = 0.364x + 69.7$

b 0.0795

c Is it not appropriate to estimate the weight of a student who does 5 hrs of exercise at the weekend because the r value is weak.

12 **a** 0.812

b Strong, positive, linear correlation?

c $y = 0.801x + 5.02$

d $y = 0.801(80) + 5.02 = 69\%$

e This is a valid estimate as it has a strong r value and we are using interpolation.

f The regression line cannot be used to estimate the score of a student who scored 10% in mathematics? as the score is outside of the given domain and extrapolation is unreliable.

g $x = 0.823y + 23.5$

h This equation be used to estimate a mathematics score, given a science score.

13 **a** Random sampling

b 0.295

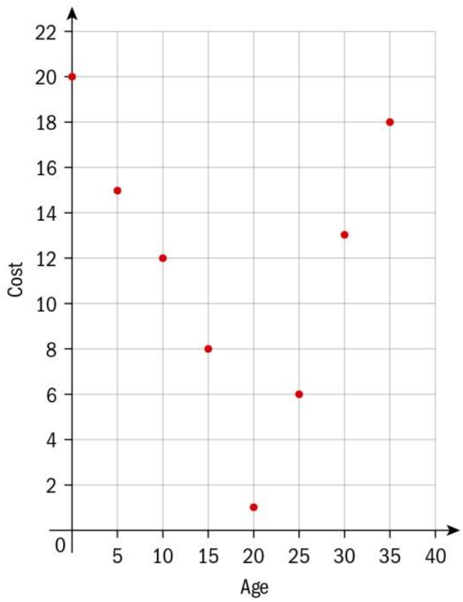
c As the age of a person increases, so does the number of shoes they have.

d $y = 0.0421x + 4.28$

e $y = 0.0421(18) + 4.28 = 4.62 or 5 pairs$ of shoes

f No. The r value is weak.

14 a



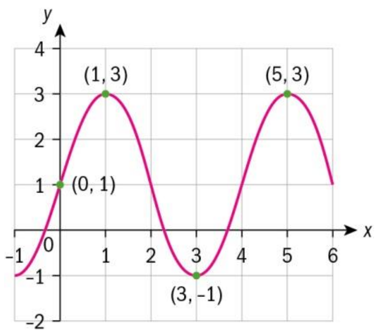
b No. The data cannot be represented by one line but two lines would be appropriate.

c Using the calculator to find a function from $x = 0$ to $x = 20$ gives $y = -0.9x + 20.2$ and then using the calculator to find a function from $x = 20$ to $x = 35$ gives $y = 1.16x - 22.4$. The two functions are $y = -0.9x + 20.2, 0 \le x \le 20$ and $y = 1.16x - 22.4, 20 \le x \le 35$.

6 Relationships in space: geometry and trigonometry

Section A. A calculator is not allowed

1 The diagram shows the graph of the function *f* given by $f(x)=p\sin+q$,



Find the values of

a the period **b** p **c** q.

2 **a** Show that $\cot\theta=\dfrac{1+\cos2\theta}{\sin2\theta}$

b Hence find the **EXACT** value of $\cot\dfrac{\pi}{8}$

3 The angle θ satisfies the equation $2\tan^2\theta-5\sec\theta-10=0$, where θ acute. Find the value of sec θ.

4 If $\sin(x-a)=b\sin(x+a)$ express $\tan x$ in terms of a and b.

Section B. A calculator is allowed

5 A metal sphere of volume $3000cm^3$ is melted down and recast into a solid cone of base radius 8 cm. Find

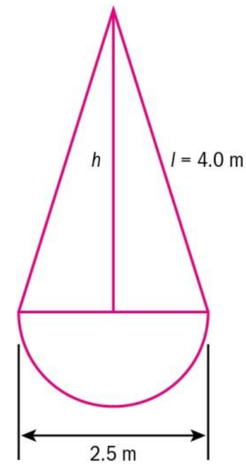
a The radius of the sphere.

b The perpendicular height of the cone, assuming that 15% of the metal is lost in the process.

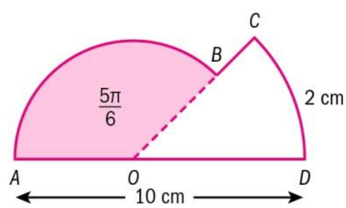
6 A harbor buoy is made from a cone on top of a hemisphere. The diameter of the cone and hemisphere is 2.5 m and the slant height of the cone (l) is 4 m. Find

a the volume and

b surface area of the buoy.



7 A semi-circle with centre O is partly covered with sector OCD. Given that angle $AOB = \frac{5\pi}{6}$, find the size of the shaded area.



8 **a** Find the smallest angle in a triangle of sides 5m, 7m, 8m.

b Find the area of the triangle

9 In triangle ABC, $AB = 20cm$, $AC = 17cm$ and $\angle B = 50°$. Calculate the two possible values for angle C.

10 The depth (d) of water at the end of a pier on Tuesday is modelled by

$$d(h) = 4\sin h\left(\frac{\pi}{6}\right) + 8$$

where h is the number of hours after midnight.

a Find the maximum depth of water.

b Sketch the graph showing the height of water for Tuesday and draw a line on the sketch to show the times between which there is 10m or more of water at the end of the pier. State these times (in the form am and/or pm) under your sketch.

Answers

1 **a** $Period = 5 - 1 = 4$

b $Amplitude = p = \dfrac{3-(-1)}{2} = 2$

c $Vertical\,shift = q = \dfrac{3+(-1)}{2} = 1$

2 **a** $\dfrac{1+cos2\theta}{sin2\theta} = \dfrac{1+2\cos^2\theta - 1}{2sin\theta cos\theta} = \dfrac{2\cos^2\theta}{2sin\theta cos\theta} = \dfrac{cos\theta}{sin\theta} = cot\theta$

b $cot\dfrac{\pi}{8} = \dfrac{1+cos\frac{\pi}{4}}{\sin\frac{\pi}{4}} = \dfrac{1+\sqrt{2}/2}{\sqrt{2}/2} = 1+\sqrt{2}$

3 $2tan^2\theta - 5sec\theta - 10 = 0$ $1 + tan2\theta = sec2\theta$,

$2(sec^2\theta - 1) - 5sec\theta - 10 = 0$

$2sec^2\theta - 5sec\theta - 12 = 0$

$(2sec\theta + 3)(sec\theta - 4) = 0$ $sec\theta = -3$ *or* $sec\theta = 4$

θ is acute means θ is positive.

$sec\theta = 4$

4 $sinx\,cosa - cosx\,sina = b\,sinx\,cosa + b\,cosx\,sina$ $tanx\,cosa - sina = b\,tanx\,cosa + b\,sina$

$\tan x = -\dfrac{(b+1)\sin a}{(b-1)\cos a} = -\left(\dfrac{b+1}{b-1}\right)\tan a$

5 **a** $r = \sqrt[3]{\dfrac{3\times 3000}{4\pi}} = 8.95cm$

b Volume of the cone $= \dfrac{1}{3}\pi(8^3)h = 3000\times 0.85 = 2550$

$h = \dfrac{2550\times 3}{64\pi} = 38.0cm$

6 **a** $h = \sqrt{4^2 - (1.25)^2} = 3.80m$

Total volume = volume of the hemisphere + volume of the cone

$V = \dfrac{2}{3}\pi r^3 + \dfrac{1}{3}\pi r^2 h = \dfrac{2}{3}\pi(1.25)^3 + \dfrac{1}{3}\pi(1.25)^2(3.80) = 10.3m^3$

b Total surface area = surface area of the hemisphere + curved surface area of the cone

$TSA = 2\pi r^2 + \pi rl = 2\pi(1.25)^2 + \pi(1.25)(4) = 25.5m^2$

7 In sector OAD $l = r\theta$, $\theta = \frac{\pi}{6}$

$$2 = r \times \frac{\pi}{6}$$

$$r = \frac{12}{\pi}$$

In sector OAB

$$r = 10 - \frac{12}{\pi} = 3.82cm$$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2} \times \left(10 - \frac{12}{\pi}\right)^2 \left(\frac{5\pi}{6}\right) = 19.1cm^2$$

8 **a** $cos\theta == =0.7857$

θ = 38.2°

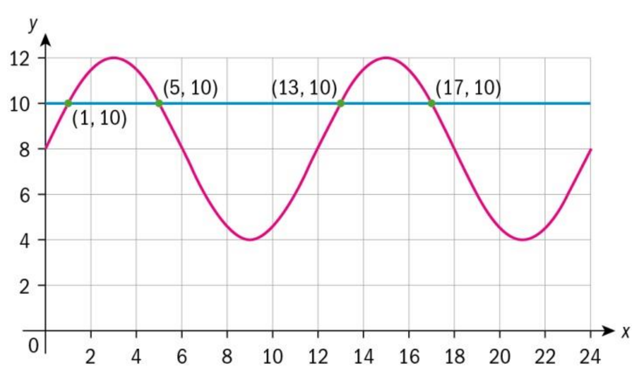
b $Area = \frac{1}{2} \times 8 \times 7 \times sin38.2° = 17.3cm2$

9 $\frac{\sin C}{20} = \frac{\sin 50°}{17}$

$$\sin C = \frac{20\sin 50°}{17} = 0.901$$

$$\angle C = 64.3°, 116°$$

10 **a** Maximum depth when $h\left(\frac{\pi}{6}\right) = 1$, $d(h) = 4(1) + 8 = 12m$



b 1am to 5am and 1pm to 5pm

7 Generalizing relationships: exponents, logarithms and integration

Section A. A calculator is not allowed

1 Find the value of $\log_x x^2 + \log_x\left(\frac{1}{x}\right)$

2 Solve

a $9^x - 7(3^x) = 18$

b $4^{x+1} = \frac{1}{8^{x-2}}$

3 Solve

a $\log 2 + 2\log x = \log(5x+3)$

b $\log_3(x+34) - 2 = \log_3 2x$

c $\log_2 x + \log_4 x + \log_{16} x - 14 = 0$

4 A curve has equation $y = e^{2x}$, find the coordinates of the point on the curve $y = e^{2x}$, where the gradient of the curve is $\frac{1}{2}$. Give your answer in the form $-\ln a$, where $a \in \mathbb{Z}$

5 Find the equation of the tangent to $y = \tan x$ at the point where $x = \frac{\pi}{4}$

6 **a** Use the substitution $u = x+2$ to find $\int \frac{x}{(x+2)^2} dx$

b Use integration by parts to find $\int x\cos x\, dx$

7 Find $\int_0^{\frac{\pi}{4}} \frac{\sin x}{\sqrt{\cos x}} dx$.

8 The function f is defined on the domain $x \geq 0$ by $f(x) = \frac{x^2}{e^x}$.

a Find the maximum value of $f(x)$, and justify that it is a maximum.

b Find the x coordinates of the points of inflexion on $f(x)$.

Section B. A calculator is allowed

9 When a course ends, students start to forget the material they have learned. The Ebbinghaus model (sometimes called the forgetting curve) assumes the percentage (*p*) of knowledge that a student retains after time, *t* weeks, is related by

$$p = (100 - a)e^{-bt} + a$$

where *a* and *b* are traits of an individual. If Chike has an *a* value of 20 and a *b* value of 0.3, how much information will Chike retain after 4 weeks?

10 Solve the following equations to find the value of *x* to 3 significant figures

a $4^x = 9$

b $e^{3x} - 5^{1-x} = 0$

c Solve $6^x = 3^{x+1}$ giving your answer in the form $\dfrac{\ln a}{\ln b}$ where *a*, *b* are integers.

11 Given $f(x) = x\ln(4 - x^2)$

a Find $f'(x)$

b Sketch the curve within the domain $-2 \le x \le 2$.

c Hence write down the solutions of $f'(x) = 0$.

12 Find the area of the region completely enclosed by the curve $y = e^{-x} - x + 1$ and the *x* axes.

Answers

1 $2+(=1)=1$

2 **a** $3^{2x}-7(3^x)-18=0$

$(3^x)^2-7(3^x)-18=0$

$(3^x+2)(3^x-9)=0$

$3^x=-2$ *or* $3^x=9$

$x=2$

b $4^{x+1}=\frac{1}{8^{x-2}}$

$2^{2x+2}=2^{6-3x}$

$2x+2=6-3x$

$x=\frac{4}{5}$

3 **a** $\log 2+2\log x=\log(5x+3)$

$log\,2x^2=log\,(5x+3)$

$2x^2=5x+3$

$2x^2-5x-3=0$

$(2x+1)(x-3)=0$

$x=3$

b $\log_3(x+34)-2=\log_3 2x$

$\log_3\left(\frac{x+34}{2x}\right)=2$

$\frac{x+34}{2x}=9$

$x+34=18x$

$x=2$

c $\log_2 x+\log_4 x+\log_{16} x=14$

$$\log_2 x + \frac{\log_2 x}{\log_2 4} + \frac{\log_2 x}{\log_2 16} = 14$$

$$\log_2 x + \frac{\log_2 x}{2} + \frac{\log_2 x}{4} = 14$$

$$4\log_2 x + 2\log_2 x + \log_2 x = 56$$

$$7\log_2 x = 56$$

$$\log_2 x = 8$$

$$x = 256$$

4 $f'(x) = 2e^{2x} = \frac{1}{2}$

$$e^{2x} = \frac{1}{4}$$

$$\ln e^{2x} = \ln\frac{1}{4}$$

$$2x = ln\frac{1}{4}$$

$$x = \frac{1}{2}\ln\frac{1}{4} = \ln\frac{1}{2} = -\ln 2$$

5 $f'(x) = \frac{1}{\cos^2 x}$

$$f'\left(\frac{\pi}{4}\right) = \frac{1}{\cos^2\left(\frac{\pi}{4}\right)} = \frac{1}{\left(\frac{\sqrt{2}}{2}\right)^2} = 2 \quad \text{at } \left(\frac{\pi}{4}, \tan\frac{\pi}{4}\right) \text{or} \left(\frac{\pi}{4}, 1\right)$$

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 2\left(x - \frac{\pi}{4}\right)$$

$$y = 2x + \left(1 - \frac{\pi}{2}\right)$$

6 a

$$\int \frac{x}{(x+2)^2}dx = \int \frac{(u-2)^3}{u^2}du = \int \frac{u^3 - 6u^2 + 12u - 8}{u^2}du$$

$$= \int u du + \int (-6) du + \int \frac{12}{u}du - \int 8u^{-2}du$$

$$= \frac{u}{2} - 6u + 12\ln|u| - 8u^{-1} + c$$

$$= \frac{(x+2)^2}{2} - 6(x+2) + 12\ln|x+2| - \frac{8}{x+2} + c$$

b $u = x, \quad \frac{du}{dx} = 1$

$$\frac{dv}{dx} = \sin x, \quad v = \cos x$$

$$\int x cox\, x\, dx = x sin x - \int \sin x\, dx = x \sin x + \cos x + c$$

7 Let $u = cosx$

$$\frac{du}{dx} = -\sin x$$

when $x = \frac{\pi}{4}, u = \frac{1}{\sqrt{2}}$; when $x = 0$, $u = 1$

$$\int_0^{\frac{\pi}{4}} \frac{\sin x}{\sqrt{\cos x}} dx = \int_1^{\frac{1}{\sqrt{2}}} -\frac{1}{u^{\frac{1}{2}}} du = \int_1^{\frac{1}{\sqrt{2}}} -u^{\frac{1}{2}} du = \left[-2u^{\frac{1}{2}}\right]_1^{\frac{1}{\sqrt{2}}} = -\frac{2}{2^{\frac{1}{4}}} + 2 = 2 - 2\frac{3}{4}$$

8 **a** $f'(x) = \frac{2xe^x - x^2e^x}{e^{2x}} = \frac{x(2-x)}{e^x}$

For a maximum $f'(x) = 0 \qquad x(2-x) = 0$

$x = 0$ or 2

$$f''(x) = \frac{(2-2x)e^x - e^x(x(2-x))}{e^{2x}} = \frac{x^2 - 4x + 2}{e^x}$$

$f''(0) = 2 > 0$ minimum

$f''(2) = -\frac{2}{e^2} < 0$ maximum

Maximum value $f(2) = \frac{4}{e^2}$

b $f''(x) = \frac{x^2 - 4x + 2}{e^x} = 0$

$x = 2 \pm \sqrt{2}$

9 $p = (100 - 20)e^{-(0.3\times 4)} + 20 = 44.1\%$

10 a $log4^x = log9$

$x log4 = log9$

$$x = \frac{log9}{log4} = 1.58$$

b $e^{3x} - 5^{1-x} = 0$

$e^{3x} = 5^{1-x}$

$lne^{3x} = ln5^{1-x}$

$3xlne = (1-x)ln5$

$3x + xln5 = ln5$

$x(3 + ln5) = ln5$

$x = \dfrac{ln5}{3 + ln5} = 0.349$

c $\ln 6^x = \ln 3^{x+1}$

$x \ln 6 = (x+1)\ln 3$

$x \ln 6 = x \ln 3 + \ln 3$

$x \ln 6 - x \ln 3 = \ln 3$

$x(\ln 6 - \ln 3) = \ln 3$

$x = \dfrac{\ln 3}{(\ln 6 - \ln 3)}$

$x = \dfrac{\ln 3}{\ln 2}$

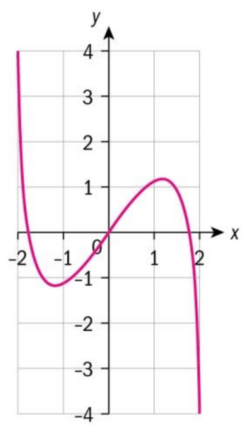
11 a $f'(x) = uv' + vu'$

$u = 4 - x^2 \quad y = lnu \; \dfrac{du}{dx} = -2x, \; \dfrac{dy}{du} = \dfrac{1}{u} = \dfrac{1}{4 - x^2} \; \dfrac{dy}{dx} = \dfrac{dy}{du} \times \dfrac{du}{dx} \; \dfrac{dy}{dx} = \dfrac{-2x}{4 - x^2}$

$u = x \; v = \ln(4 - x^2) \; u' = 1 v' = \dfrac{-2x}{4 - x^2} \; f'(x) = \left(x \times \dfrac{-2x}{4 - x^2}\right) + \ln(4 - x^2)$

$f'(x) = \left(\dfrac{-2x^2}{4 - x^2}\right) + \ln(4 - x^2)$

b



c $x = -1.15, 1.15$

12 Area = 1.18

8 Modelling changes: more calculus

Section A. A calculator is not allowed

1 Find the area bounded by $f(x) = -x^2 - 2x$ and $g(x) = x$

2 Find the exact volume obtained when the portion of the curve $y = x^2$ between $x = 0$ and $x = 2$ is rotated around the x axis.

3 The velocity of a particle is modelled by $v(t) = 3t^2 + 5t$, where t is measured in seconds. Find the displacement of the particle after 5 seconds

4 Solve the separable differential equations.

a $\frac{x^2}{y} = 4y\frac{dy}{dx}$, given $x = 2$ when $y = 1$

b $x\frac{dy}{dx} - y^2 = 1$ given $x = 2$ when $y = 0$

c $2x\frac{dy}{dx} = 1 + y^2$ given $x = 1$ when $y = 1$

5 Solve $x\frac{dy}{dx} = x + 2y$ when $x = 1$ and $y = 2$ by using the substitution $y = cx$

6 Find the general solution to $\frac{dy}{dx} - \frac{3y}{x+1} = (x-1)^4$

7 Use L'Hopital's rule, if possible, to find $\lim_{x \to 0} \frac{\sin x}{x}$

8 **a** Find the first three terms of the Maclaurin series for $ln(1+ex)$.

b Use L'Hopital's rule to find the value of $\lim_{x \to 0} \frac{2\ln(1+e^x) - x - \ln 4}{x^2}$.

Section B. A calculator is allowed

9 $f(x) = x^3 - 9x$ and $g(x) = -2x + 6$

a Sketch the graph of $f(x)$ and $g(x)$ on the same axes with $-4 \le x \le 4$ and shade the areas enclosed between the two functions.

b Write down the points of intersection.

c Find the area enclosed between $f(x)$ and $g(x)$.

10 Find the area bounded the curves $f(x) = 4x + 3 - x^2$ and $g(x) = -x^3 + 7x^2 - 10x + 5$ and the lines $x = 1$ and $x = 2$.

11 Find the volume of the solid obtained by rotating the region bounded by $y = 2\sqrt{x-1}$ and $y = x - 1$ around $y = 0$.

12 Find the volume obtained when the portion of $y^2 = 4x$ between $(0,0)$ and $(1,2)$ is rotated around the y axis.

Answers

1 Find the x value at the point of intersection

$$-x^2-2x=x$$
$$x^2+3x=0$$
$$x=0,-3$$

$$\text{Area} = \int_{-3}^{0}(-x^2-2x)-(x)dx = \int_{-3}^{0}(-x^2-3x)dx$$

$$\left[-\frac{x^3}{3}-\frac{3x^2}{2}\right]_{-3}^{0} = 0-\frac{(-3)^3}{3}-\frac{3(-3)^2}{2}=\frac{9}{2}$$

2 $V=\pi\int_0^2 x^4dx=\pi\left[\frac{1}{5}x^5\right]_0^2=\pi(\frac{1}{5}(2)^5-0)=\frac{32\pi}{5}$

3 The distance travelled, s, is the area under the velocity-time graph.

$$s(t)=\int_0^5 v(t)=\int_0^5(3t^2+5t)dt=\left[t^3-\frac{5}{2}t^2\right]_0^5=125+\frac{125}{2}=\frac{375}{2}=187.5m$$

4 **a** $x^2dx=4y^2dy$

$$\int x^2dx=\int 4y^2dy$$
$$\frac{x^3}{3}=\frac{4y^3}{3}+\frac{4}{3}$$
$$4y^3=x^3-4$$
$$y=\sqrt[3]{\frac{x^3}{4}-1}$$

b $x\frac{dy}{dx}=y^2+1$

$$\frac{dy}{y^2+1}=\frac{dx}{x}$$
$$\arctan y=\ln x+c$$
$$\arctan 0=\ln 2+c$$
$$c=-\ln 2$$
$$\arctan y=\ln x-\ln 2=\ln\frac{x}{2}$$
$$y=\tan\left(\ln\frac{x}{2}\right)$$

c $\frac{1}{y^2+1}\frac{dy}{dx}=\frac{1}{2x}$

$$\int\frac{1}{y^2+1}\frac{dy}{dx}dx=\int\frac{1}{2x}dx$$
$$\arctan y=\frac{1}{2}\ln x+c$$
$$y=\tan\frac{1}{2}\ln x+c$$

$$1 = \tan\frac{1}{2}\ln 1 + c$$
$$\tan c = 1$$
$$c = \frac{\pi}{4}$$
$$y = \tan\frac{1}{2}\ln x + \frac{\pi}{4}$$

5 Solve by using the substitution $y = cx$

$$x\frac{dy}{dx} = x + 2y \text{ when } x = 3 \text{ and } y = \frac{3}{2}\text{; substitute}$$
$$\frac{dy}{dx} = c + x\frac{dc}{dx}$$
$$c + x\frac{dc}{dx} = \frac{x + 2cx}{x}$$
$$c + x\frac{dc}{dx} = 1 + 2c$$
$$x\frac{dc}{dx} = 1 + c$$
$$\frac{dc}{dx} = \frac{1 + c}{x}$$

$$\int\frac{1}{1+c}dc = \int\frac{1}{x}dx$$
$$\ln 1 + c = \ln x + c$$
$$\text{When} c = lnA, lc1 + c = lnAx$$
$$1 + c = Ax$$
$$v = \frac{y}{x}\text{, therefore } 1 + \frac{y}{x} = Ax\text{, and } \mathrm{y} = \mathrm{Ax}^2 - x$$
$$2 = A - 1$$
$$A = 3$$
$$y = 3x^2 - x$$

6 $\dfrac{dy}{dx} - \dfrac{3y}{x+1} = (x-1)^4$

$$I = e^{\int P dx} = e^{\int\frac{-3}{x+1}dx}$$

$$\int\frac{-3}{x+1}dx = -3\ln(x+1) = \ln(x+1)^{-3} = \ln\frac{1}{(x+1)^3}$$

$$I = e^{\ln\frac{1}{(x+1)^3}} = \frac{1}{(x+1)^3}$$

$$\frac{1}{(x+1)^3}\left(\frac{dy}{dx} - \frac{3y}{x+1}\right) = \frac{1}{(x+1)^3}(x-1)^4$$

$$\frac{1}{(x+1)^3}\frac{dy}{dx} - \frac{3y}{(x+1)^4} = x + 1$$

Integrate both sides to give

$$\frac{y}{(x+1)^3} = \frac{1}{2}x^2 + x + c$$

$$y = (x+1)^3\left(\frac{1}{2}x^2 + x + c\right)$$

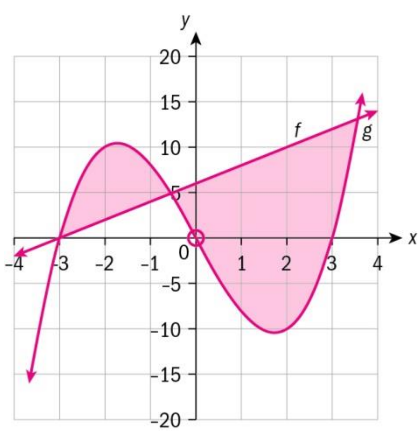
7 $\lim\limits_{x\to 0}\frac{\sin x}{x} = \frac{\lim\limits_{x\to 0}\frac{d}{dx}(\sin x)}{\lim\limits_{x\to 0}\frac{d}{dx}(x)} = \frac{\lim\limits_{x\to 0} cosx}{\lim\limits_{x\to 0} 1} = \frac{1}{1} = 1$

8 **a** $ln(1+e^x)=ln(1+1+x+\ ...) =$

$$= \ln 2 + \left(\frac{1}{2}x + \frac{1}{4}x^2\right) - \frac{1}{2}\left(\frac{1}{2}x + \frac{1}{4}x^2\right) = \ln 2 + \frac{1}{2}x + \frac{1}{8}x^2$$

b $\lim\limits_{x\to 0}\frac{2\ln(1+e^x) - x - \ln 4}{x^2} = \lim\limits_{x\to 0}\frac{2e^x \div (1+e^x) - 1}{2x} = \lim\limits_{x\to 0}\frac{2e^x \div (1+e^x)^2}{2} = \frac{1}{4}$

9 **a**



b $(-3,0)(-0.562,4.88)(3.56,13.1)$

c $\int_{-3}^{-0.562} x^3 - 7x - 6dx + \int_{-0.562}^{3.56} -x^3 + 7x + 6dx = 32.3$ square units

10 $\int_1^2 x^3 - 8x^2 + 14x - 2dx = 12.4$square units

11 $Volume = \pi\int_1^5 \left(\left(2\sqrt{x-1}\right) - (x-1)\right)^2 dx = 76.6$cubic units

12 $Volume = \pi\int_0^2 x^2 dy = \pi\int_0^2 \frac{y^4}{16}dy = 1.26$ cubic units

9 Modelling 3D space: vectors

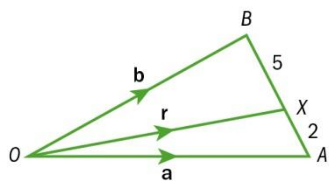
Section A. A calculator is not allowed

1 Given that the point P has position vector $3\boldsymbol{i}-11\boldsymbol{j}$ and the point Q has position vector $8\boldsymbol{i}+\boldsymbol{j}$,

a Find $\overrightarrow{PQ}$

b Find $\left|\overrightarrow{PQ}\right|$

2 In triangle OAB, write $\boldsymbol{r}$ in terms of $\boldsymbol{a}$ and $\boldsymbol{b}$.



3 Find a unit vector, $\boldsymbol{v}$, in the same direction as $6\boldsymbol{i}+8\boldsymbol{j}$

4 Prove that the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ are collinear if these are given as $P(4,1,3), Q(8,4,6) \text{and} R(20,13,15)$.

5 Show that $\boldsymbol{a}$ and $\boldsymbol{b}$ are perpendicular when $\boldsymbol{a}=\begin{pmatrix} 2\cos\frac{\pi}{4} \\ 2\sin\frac{\pi}{4} \end{pmatrix}$ and $\boldsymbol{b}=\begin{pmatrix} -3\sin\frac{\pi}{4} \\ 3\cos\frac{\pi}{4} \end{pmatrix}$.

6 Find

a the vector equation

b the Cartesian equation

of the line which passes through the points $A\ (1,2)$ and $B\ (-3,4)$.

7 Find the equation of the plane containing the points $A(3,2,3), B(1,4,2) \textit{and} C(1,5,3)$

8 The points $A(1,2,1), B(-3,1,4), C(5,-1,2)$ lie on the same plane.

Find the vectors

a $\overrightarrow{AB}$ **b** $\overrightarrow{AC}$

c Find the Cartesian equation of the plane.

d Find the vector equation of the line that passes through $D(5,3,7)$ and is perpendicular to the plane.

Section B. A calculator is allowed

9 Find the angle between $\begin{pmatrix}1\\2\end{pmatrix}$ and $\begin{pmatrix}6\\-8\end{pmatrix}$.

10 Find the angle between the planes $3x+2y+z=0$ and $x+2y+4=0$

Answers

1 **a** $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (8\boldsymbol{i} + \boldsymbol{j}) - (3\boldsymbol{i} - 11\boldsymbol{j}) = 5\boldsymbol{i} + 12\boldsymbol{j}$

b $\left|\overrightarrow{PQ}\right| = \sqrt{5^2 + 12^2} = 13$

2 $\boldsymbol{r} = \overrightarrow{OX} = \overrightarrow{OA} + \overrightarrow{AX} = \overrightarrow{OA} + \frac{2}{7}\overrightarrow{AB} = \boldsymbol{a} + \frac{2}{7}(\boldsymbol{b} - \boldsymbol{a}) = \frac{5}{7}\mathbf{a} + \frac{2}{7}\boldsymbol{b}$

3 $|6\boldsymbol{i} + 8\boldsymbol{j}| = \sqrt{6^2 + 8^2} = 10$

$\boldsymbol{v} = \frac{1}{10}(6\boldsymbol{i} + 8\boldsymbol{j}) = \frac{3}{5}\boldsymbol{i} + \frac{4}{5}\boldsymbol{j}$

4 $\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = (8\boldsymbol{i} + 4\boldsymbol{j} + 6\boldsymbol{k}) - (4\boldsymbol{i} + \boldsymbol{j} + 3\boldsymbol{k}) = 4\boldsymbol{i} + 3\boldsymbol{j} + 3\boldsymbol{k}$

$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ} = (20\boldsymbol{i} + 13\boldsymbol{j} + 15\boldsymbol{k}) - (8\boldsymbol{i} + 4\boldsymbol{j} + 6\boldsymbol{k}) = 12\boldsymbol{i} + 9\boldsymbol{j} + 9\boldsymbol{k}$

$\overrightarrow{QR} = 3\overrightarrow{PQ}$. Therefore they are collinear

5 $\boldsymbol{r} \cdot \boldsymbol{s} = \left(2\cos\frac{\pi}{4} \times -3\sin\frac{\pi}{4}\right) + \left(2\sin\frac{\pi}{4} \times 3\cos\frac{\pi}{4}\right) = \left(\sqrt{2} \times -3\frac{\sqrt{2}}{2}\right) + \left(\sqrt{2} \times 3\frac{\sqrt{2}}{2}\right) = 0$

Therefore the vectors are perpendicular

6 **a** $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (-3\boldsymbol{i} + 4\boldsymbol{j}) - (1\boldsymbol{i} + 3\boldsymbol{j}) = -4\boldsymbol{i} + 2\boldsymbol{j}$

$\boldsymbol{r} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

b $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + t\begin{pmatrix} -4 \\ 2 \end{pmatrix}$

$x = 1 - 4t$

$y = 2 + 2t$

$t = \frac{1 - x}{4}$ and $t = \frac{y - 2}{2}$

$\frac{y - 2}{2} = \frac{1 - x}{4}$

$2y - 4 = 1 - x$

$x + 2y - 5 = 0$

7 $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = (\boldsymbol{i} + 4\boldsymbol{j} + 2\boldsymbol{k}) - (3\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) = -2\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}$

$\overrightarrow{AC} = \overrightarrow{OC} - \overrightarrow{OA} = (\boldsymbol{i} + 5\boldsymbol{j} + 3\boldsymbol{k}) - (3\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) = -2\boldsymbol{i} + 3\boldsymbol{j}$

A vector equation for the plane is

$$\boldsymbol{r} = \overrightarrow{OA} + a\overrightarrow{AB} + b\overrightarrow{AC} = (3\boldsymbol{i} + 2\boldsymbol{j} + 3\boldsymbol{k}) + a(-2\boldsymbol{i} + 2\boldsymbol{j} - \boldsymbol{k}) + b(-2\boldsymbol{i} + 3\boldsymbol{j})$$

8 **a** $\overrightarrow{AB} = \begin{pmatrix} -4 \\ -1 \\ 3 \end{pmatrix}$ **b** $\overrightarrow{AC} = \begin{pmatrix} 4 \\ -3 \\ 1 \end{pmatrix}$

c $\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ -4 & -1 & 3 \\ 4 & -3 & 1 \end{vmatrix} = \begin{pmatrix} 8 \\ 16 \\ 16 \end{pmatrix}$

Normal $\boldsymbol{n} = \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

$$\boldsymbol{r} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$$

$x + 2y + 2z = 7$

d $\boldsymbol{r} = \begin{pmatrix} 5 \\ 3 \\ 7 \end{pmatrix} + t\begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}$

$1(5+t)+2(3+2t)+2(7+2t)=7$

$9t = -18$

$t = -2$

Note that $t = -\frac{1}{4}$ if $\begin{pmatrix} 8 \\ 16 \\ 16 \end{pmatrix}$ is used.

9 $\cos\theta = \dfrac{a \cdot b}{|a||b|} = \dfrac{\begin{pmatrix} 1 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 6 \\ -8 \end{pmatrix}}{\left|\begin{pmatrix} 1 \\ 2 \end{pmatrix}\right|\left|\begin{pmatrix} 6 \\ -8 \end{pmatrix}\right|} = -\dfrac{1}{\sqrt{5}}$

$\theta = 117°$

10 The direction of the normals are $3\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}$ and $\boldsymbol{i} + 2\boldsymbol{j}$

$$Cos\theta = \frac{(3\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}) \cdot (\boldsymbol{i} + 2\boldsymbol{j})}{|3\boldsymbol{i} + 2\boldsymbol{j} + \boldsymbol{k}||\boldsymbol{i} + 2\boldsymbol{j}|} = \frac{7}{\sqrt{70}}$$

$\theta = 33.2°$

10 Equivalent systems of representation: more complex numbers

A calculator is not allowed.

1 **a** Write $z=-1-i$ in polar form and sketch point z in the coordinate system.

b Write $\sqrt{3}cis\left(\frac{\pi}{6}\right)$ in Cartesian form.

2 Find the modulus and the argument of the complex number $\frac{-9+3i}{1-2i}$.

3 Find the complex number z satisfying the equation $(3-4i)z-(1+i)z^*=13+2i$

4 Express $z=-\sqrt{3}+i$ in the form $r(\cos\theta+i\sin\theta)$, where $-\pi\le\theta\le\pi$

5 Evaluate $e^{-i\frac{\pi}{4}}$

6 Find the exact value of $\left(\sqrt{3}+i\right)^7$

7 **a** Use de Moivre's theorem to find the roots of the equation $z^4=1-i$.

b If z_1 is the root in the first quadrant and z_2 is the root in the second quadrant, find $\frac{z_2}{z_1}$ in the form $a+bi$.

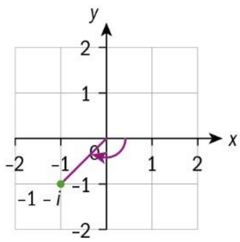
8 Solve the equation $z^3=2+2i$

Answers

1 **a** $|z| = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

$\tan^{-1} 1 = \frac{\pi}{4} and -\frac{3\pi}{4}$

$z = \left(\sqrt{2}, -\frac{3\pi}{4}\right)$



b $\sqrt{3}cis\left(\frac{\pi}{6}\right) = \sqrt{3}\left(\cos\left(\frac{\pi}{6}\right) + isin\left(\frac{\pi}{6}\right)\right) = \sqrt{3}\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = \frac{3}{2} + \frac{1}{2}i$

2 $\frac{-9+3i}{1-2i} = \frac{(-9+3i)(1+2i)}{(1-2i)(1+2i)} = \frac{-9-18i+3i-6}{1+2i-2i+4} = \frac{-15-15i}{5} = -3-3i$

$|-3-3i| = \sqrt{(-3)^2 + (-3)^2} = \sqrt{18} = 3\sqrt{2}$

$\arg(-3-3i) = (\tan^{-1} 1) - \pi = \frac{\pi}{4} - \pi = -\frac{3\pi}{4}$

3 $z = (a+ib), z^* = (a-ib)$

$(3-4i)(a+ib) - (1+i)(a-ib) = 13+2i$

$3a - 4ia + 3ib - 4i^2b - a - ia + ib + i^2b = 13+2i$

$2a + 3b + i(-5a + 4b) = 13 + 2i$

$2a + 3b = 13$

$-5a + 4b = 2$

$a = 2, b = 3$

$z = 2 + 3i$

4 $r = \sqrt{(-3)^2 + 1^2} = 2$

$\theta = \arg z = \pi - \tan^{-1}\frac{1}{\sqrt{3}} = \frac{5\pi}{6}$

$$z = 2\left(\cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6}\right)$$

5 $e^{-i\frac{\pi}{4}} = \cos\left(-\frac{\pi}{4}\right) + i\sin\left(-\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} - i\frac{\sqrt{2}}{2}$

6 $\left(\sqrt{3}+i\right)^7$

$$\left|\sqrt{3}+i\right| = \sqrt{\left(\sqrt{3}\right)^2 + 1^2} = 2$$

$$\sqrt{3}+i = 2\left(\frac{\sqrt{3}}{2} + \frac{1}{2}i\right) = 2cis\left(\frac{\pi}{6}\right)$$

$$\left(\sqrt{3}+i\right)^7 = \left(2cis\left(\frac{\pi}{6}\right)\right)^7 = 2^7 cis\left(\frac{7\pi}{6}\right) = 128\left(-\frac{\sqrt{3}}{2} - \frac{1}{2}i\right) = -64\sqrt{3} - 64i$$

7 **a** $z = (1-\mathrm{i})^{\frac{1}{4}}$

Let 1 − i = $r(\cos\theta + \mathrm{i}\sin\theta)$

$r = \sqrt{2}; \theta = -\frac{\pi}{4}$

$$z = (\sqrt{2}(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4})))^{\frac{1}{4}} = (\sqrt{2}(\cos(-\frac{\pi}{4} + 2n\pi) + i\sin(-\frac{\pi}{4} + 2n\pi)))^{\frac{1}{4}}$$

$$= 2^{\frac{1}{8}}(\cos(-\frac{\pi}{16} + \frac{n\pi}{2}) + i\sin(-\frac{\pi}{16} + \frac{n\pi}{2}))$$

$$= 2^{\frac{1}{8}}(\cos(-\frac{\pi}{16}) + i\sin(-\frac{\pi}{16}))$$

$$= 2^{\frac{1}{8}}(\cos(\frac{7\pi}{16}) + i\sin(\frac{7\pi}{16}))$$

$$= 2^{\frac{1}{8}}(\cos(\frac{15\pi}{16}) + i\sin(\frac{15\pi}{16}))$$

$$= 2^{\frac{1}{8}}(\cos(-\frac{9\pi}{16}) + i\sin(-\frac{9\pi}{16}))$$

b $\frac{z_2}{z_1} = \frac{2^{\frac{1}{8}}((\cos(\frac{15\pi}{16}) + i\sin(\frac{15\pi}{16}))}{2^{\frac{1}{8}}((\cos(\frac{7\pi}{16}) + i\sin(\frac{7\pi}{16}))} = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2} = i$

8 $z^3 = 2 + 2i$

$$|2+2i| = \sqrt{2^2 + 2^2} = \sqrt{8}$$

$$\arg(2+2i) = \tan^{-1}1 = \frac{\pi}{4}$$

$$2 + 2i = \sqrt{8}e^{\frac{\pi i}{4}}$$

$$z^3 = \sqrt{8}e^{\left(\frac{\pi i}{4}+2k\pi i\right)}$$

$$z = \sqrt{2}e^{\left(\frac{\frac{\pi i}{4}+2k\pi i}{3}\right)} = \sqrt{2}e^{\frac{(1+8k)\pi i}{12}}$$

$k = 0 \qquad z = \sqrt{2}e^{\frac{\pi i}{12}}$

$k = 1 \qquad z = \sqrt{2}e^{\frac{\pi 9i}{12}}$

$k = 2 \qquad z = \sqrt{2}e^{\frac{17\pi i}{12}}$

11 Valid comparisons and informed decisions: probability distributions

Section A. A calculator is not allowed

1 When $P(A)=0.6, P(A \cup B)=0.8$ and $P(A \mid B)=0.6$. Find $P(B)$.

2 A fund raising game at the school fair sells lottery tickets for \$4. The possible prizes are \$4, \$5, \$10, \$100 or you could not win anything. W is the amount won and is represented in this distribution.

w	0	4	5	10	100
$P(W=w)$	k	0.2	0.2	0.1	0.002

a Find the value of k

b Find how much money the fundraiser would expect to make if it sold 500 tickets.

3 A discrete random variable X has its probability distribution given by

$P(X=x)=r(x+1)$, where x is 0, 1, 2, 3, 4.

a Find *r.*

b Find $E(X)$.

4 A random variable has a probability density function given by

$$f(x)=\begin{cases} ax(2-x), & 0 \le x \le 2 \\ 0, & 0 > x > 2 \end{cases}$$

a Find *a*

b Find $E(X)$

Section B. A calculator is allowed

5 70 students come to school by bus and have an 18% chance of being late, and 65 come by car and they have a 23% chance of being late. Your friend came late, find the probability that they came by car

6 The weights of a group of students are normally distributed with mean μ and standard deviation σ. 15 % of the students weigh greater than 90kg and 12 % of them less than 40kg. Find μ and σ.

7 Mimi has a 73% chance of scoring from a free throw in basketball. What is the probability that she scores more than 7 out of ten shots?

8 A factory that makes watches finds that 2 % of them are faulty. A random sample of 100 watches are tested.

a Write down the expected number of faulty watches in the sample.

b Find the probability that three watches are faulty.

c Find the probability that more than one watch is faulty.

9 On a mathematics test, the scores were normally distributed with a mean of 74 and a standard deviation of 7.

a Sketch this on a normal curve and shade the proportion of the class would be expected to score between 60 and 80 points.

b Find the shaded area and state the percentage of the class expected to score between 60 and 80.

10 A pack of coffee is sold as weighing 230gms but the manufacturer maintains a weight of 231 grams as the average so as not to cheat customers. Coffee packs are normally distributed with a mean of 231 grams and a standard deviation of 1.5 grams.

A pack of coffee is considered to be underweight if it weighs less than 228 grams.

a What is the probability that a pack of coffee is underweight?

The manufacturer decides that the probability of a pack being underweight must be reduced to 0.002. He gives this problem to two junior executives, Eden and Haven

b Eden's suggestion is to increase the mean and leave the standard deviation unchanged. Find the value of the new mean.

c Haven's suggestion is to reduce the standard deviation and leave the mean unchanged. Find the value of the new standard deviation.

d After the probability of a pack of coffee being underweight has been reduced to 0.002, the store sells 100 packs. Find the probability that at least two of the boxes are underweight.

Answers

1 $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.8 = 0.6 + P(B) - P(A \cap B)$

$P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

$0.6P(B) = P(A \cap B)$

$0.8 = 0.6 + P(B) - 0.6P(B)$

$P(B) = 0.5$

2 **a** $k = 1 - (0.2 + 0.2 + 0.1 + 0.002) = 0.498$

b $E(W) = (0 \times 0.498) + (4 \times 0.2) + (5 \times 0.2) + (10 \times 0.1) + (100 \times 0.002) = \$3$

Profit $= 500 \times \$1 = \$500$

3 **a** $\Sigma P(X = x) = 1$

$r + 2r + 3r + 4r + 5r = 15r = 1$

$r = \dfrac{1}{15}$

b $E(X) = \dfrac{8}{3}$

4 **a** $\int_0^2 2ax - ax^2 dx = 1$

$\left[ax^2 - \dfrac{a}{3}x^3\right]_0^2 = 1$

$\dfrac{4}{3}a = 1$

$a = \dfrac{3}{4}$

b $E(X) = \dfrac{3}{4}\int_0^2 2x^2 - x^3 dx = 1$

5 Expected number of late students by bus is $0.18 \times 70 = 12.6$

Expected number of late students by car is $0.23 \times 65 = 14.95$

$P(C \mid L) == 0.543$

6 $P(X>90)=0.15$ and $P(X<40)=0.12$

$z=1.036,\ -1.175$

$1.036=\ \mu=66.6, \sigma=22.6$

7 $P(X>7)=P(X=8)+P(X=9)+P(X=10)$

$$=\binom{10}{8}(0.73)^8(0.27)^2+\binom{10}{9}(0.73)^9(0.27)^1+(0.73)^{10}=0.466$$

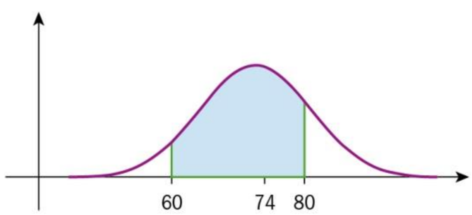
8 **a** $E(X)=100\times0.02=2$

b $P(X=3)=C_3^{100}(0.02)^3(0.98)^{97}=0.182$

c $P(X>1)=1-(P(X=0)+P(X=1))$

$P(X>1)=1-\left((0.98)^{100}+100(0.02)(0.98)^{99}\right)=0.597$

9 **a**



b Using the GDC $P(60\le s\le 80)=78.2\%$

10 **a** $X\sim N(231, 1.5^2)$

P(X < 228) = 0.0228

b $X\sim$ N(μ, 1.5^2)

P(X < 228) = 0.002

$-2.88=\dfrac{228-\mu}{1.5}$.

$\mu=232\,grams$

c $X\sim N(231,\sigma^2)$

$-2.88=\dfrac{228-231}{\sigma}$.

$\sigma=1.04\,grams$

d $X \sim B(100, 0.002)$

$P(X \le 1) = 0.982...$

$P(X \le 2) = 1 - P(X \le 1) = 0.0174$

The probability that 2 packs are underweight is 0.0174